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New results for the missing quantum numbers labelling the quadrupole and octupole boson basis

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Abstract

The many 2^k -pole boson states, $|N_k v_k \alpha_k I_k M_k\rangle$ with $k = 2, 3$, realize the irreducible representation (IR) for the group reduction chains $SU(2k+1) \supset R_{2k+1} \supset R_3 \supset R_2$. They have been analytically studied and widely used for the description of nuclear systems. However, no analytical expression for the degeneracy $d_v^{(k)}(I)$ of the R_{2k+1} IR, determined by the reduction $R_{2k+1} \supset R_3$, with $k = 2, 3$ is available. Thus, the number of distinct values taken by α_k has been so far obtained by solving some complex equations. Here we derive analytical expressions for the degeneracy $d_v^{(k)}(I)$ ($k = 2, 3$), characterizing the octupole and quadrupole boson states, respectively. The merit of this work consists of the fact that it completes the analytical expressions for the 2^k -pole boson basis for $k = 2, 3$. The general case of R_{2k+1} IR representation degeneracy is also presented and a compact analytical expression for $d_v^{(l)}(I)$ is derived.

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1. Introduction

Since the liquid drop model was discovered [1] both phenomenological and microscopic formalisms use quadrupole and octupole coordinates to describe basic properties of nuclear systems. Based on these coordinates one defines quadrupole and octupole boson operators in terms of which model Hamiltonians and transition operators are defined. Quadrupole properties for a large number of nuclei can be described by diagonalizing a quadrupole boson Hamiltonian in the basis $|N_2 v_2 \alpha_2 I_2 M_2\rangle$ associated with the irreducible representation corresponding to the group reduction chain $SU(5) \supset R_5 \supset R_3 \supset R_2$. The quantum numbers N_2 (the number of quadrupole bosons), v_2 (seniority) and I_2 are determined by the eigenvalues of the Casimir operators of the groups $SU(5)$, R_5 and R_3 , respectively. The angular momentum

projection on the z -axis is denoted by M_2 . α_2 is usually called the missing quantum number and labels the R_3 irreducible representations which are characterized by the same angular momentum I and belongs to the same irreducible representation of R_5 . The name suggests the absence of an intermediate group between R_5 and R_3 having a Casimir operator whose eigenvalues would distinguish the states of the same I_2 belonging to the same irreducible representation of R_5 , v . The $SU(5)$ boson basis has been analytically derived by three groups, following different procedures [2–5]. Despite the elegance and the strength of the methods developed in the above quoted references, no analytical solution for the number of distinct values acquired by α for a fixed pair of v_2 and I_2 , denoted by $d_{v_2}(I_2)$, was presented. Of course for each (v_2, I_2) one knows how to calculate $d_{v_2}(I_2)$ numerically, as the number of solutions (p_2) for the inequality:

$$v_2 - I_2 \leq 3p_2 \leq v_2 - \frac{1}{2}(I_2 + 3r_2), \quad r_2 = \frac{1}{2}[1 - (-1)^{I_2}]. \quad (1.1)$$

Other algorithms for calculating the multiplicity of the irreducible representations in the chain $SU(5) \supset R_5 \supset R_3 \supset R_2$ are presented in [6–8].

The octupole boson states are classified by the irreducible representations of the groups involved in the reduction chain $SU(7) \supset R_7 \supset R_3 \supset R_2$ and denoted by $|N_3 v_3 \alpha_3 I_3 M_3\rangle$. The quantum numbers are the number of octupole bosons (N_3), seniority (v_3), the missing quantum number (α_3), the angular momentum carried by the octupole bosons (I_3) and its projection on the z -axis (M_3). The octupole boson number, seniority and angular momentum are related to the Casimir operator eigenvalues associated with the groups $SU(7)$, R_7 and R_3 , respectively. The need to calculate the octupole boson states degeneracy was first met in [9, 10], where a microscopic quadrupole–octupole boson expanded Hamiltonian was treated in the basis $|N_3 v_3 \alpha_3 I_3 M_3\rangle$. Therein the degeneracy $d_{v_3}(I_3)$ was written as a contour integral which is to be performed each time for a given value of the pair (v_3, I_3) , by making use of the Cauchy theorem. Moreover, a very useful factorization of Wigner–Eckart type, for the matrix elements of octupole operators, has been presented. Later on a lengthy recursion equation for $d_{v_3}(I_3)$ was derived in [7]. The found equation was solved numerically for many (v_3, I_3) up to very high values, in [11]. Extending the harmonic function method developed in [4], from the quadrupole bosons to the octupole bosons, analytical expressions for the states $|N_3 v_3 \alpha_3 I_3 M_3\rangle$ have been derived in [12].

The study of octupole degrees of freedom in complex nuclei is an interesting subject which deserves attention from theoreticians as well as from experimentalists due to the fact that systems with static octupole deformations do not exhibit space reflection symmetry and consequently new specific properties are expected to be found.

It is worth mentioning that the embedding of R_3 in R_7 mentioned above is not unique, which results in several ways of defining a basis for octupole bosons. The basis mentioned above is used by several groups [9, 10, 13, 14]. The advantage of the provided basis consists of the fact that it has formally the same labelling as the quadrupole basis as well as the many fermion states in the seniority scheme. It is known that there is a one to one correspondence between the IR of R_7 and those of G_2 . This property has been used in [12] to build a boson basis $|N v r q s I M\rangle$ with the quantum numbers rqs , named intrinsic quantum numbers, instead of the missing quantum number α .

As mentioned before, in the past the present authors investigated analytically both the quadrupole and octupole boson bases. Here we attempt to complete our previous study and present analytical expressions for the degeneracy $d_{v_3}(I_3)$ and $d_{v_2}(I_2)$. This goal will be pursued according to the following plan. In section 2 we consider the case of octupole basis while in section 3 the quadrupole case will be treated. The general case is treated in section 4. In the last section, a short summary will be presented.

2. Degeneracy of octupole boson states

The character of an R_7 irreducible representation has the expression [15, 16]

$$\chi_{3v}(\varphi_1, \varphi_2, \varphi_3) = \frac{\det(e^{i\varphi_m K_n} - e^{-i\varphi_m K_n})_{m,n=1,2,3}}{\det(e^{i\varphi_m L_n} - e^{-i\varphi_m L_n})_{m,n=1,2,3}}, \tag{2.1}$$

where $\det(x_{m,n})_{m,n=1,2,3}$ denotes the determinant associated with the matrix $(x_{m,n})_{m,n=1,2,3}$. (L_1, L_2, L_3) is the sum of all positive roots for the group R_7 , i.e. $(L_1, L_2, L_3) = (5/2, 3/2, 1/2)$. The vector (K_1, K_2, K_3) is obtained by adding to (L_1, L_2, L_3) the highest weight (S_1, S_2, S_3) vector which for the group R_7 is equal to $(v, 0, 0)$. $(\varphi_1, \varphi_2, \varphi_3)$ is an arbitrary vector. The restriction of R_7 to R_3 can be achieved by setting

$$\frac{\varphi_1}{3} = \frac{\varphi_2}{2} = \varphi_3 = \varphi. \tag{2.2}$$

On the other hand the irreducible representation I of the group R_3 is characterized by

$$\chi_I(\varphi) = \frac{\sin(I + \frac{1}{2})\varphi}{\sin \frac{1}{2}\varphi}. \tag{2.3}$$

Let us consider the set \mathcal{C} of conjugated elements of R_3 . The complex functions defined on \mathcal{C} can be organized as a Hilbert space \mathcal{S} with the scalar product defined by

$$(f, g) = \int_0^{2\pi} f^*(\varphi)g(\varphi)\rho(\varphi) d\varphi, \tag{2.4}$$

where f and g are two elements of \mathcal{S} and ρ denotes the Haar measure for R_3 [17] whose expression is

$$\rho(\varphi) = \frac{1}{\pi} \sin^2 \frac{\varphi}{2}. \tag{2.5}$$

The set of functions $(\chi_I)_I$ is complete in \mathcal{S} and therefore any function $\chi_{3v}(\varphi)$ can be expanded as

$$\chi_{3v}(\varphi) = \sum_I d_v^{(3)}(I) \chi_I(\varphi). \tag{2.6}$$

The expansion coefficient $d_v^{(3)}(I)$ is just the multiplicity of the representation (I) characterizing the (v) representation splitting. Taking into account that χ_I are orthonormal functions one obtains

$$d_v^{(3)}(I) = \int_0^{2\pi} \chi_I^*(\varphi)\chi_{3v}(\varphi)\rho(\varphi) d\varphi. \tag{2.7}$$

Changing the integration variable from φ to $z = e^{i\varphi}$, the above equation becomes

$$d_v^{(3)}(I) = \frac{i}{4\pi} \int_{|z|=1} F(z) dz, \tag{2.8}$$

$$F(z) = \frac{(z^{v+1} - 1)(z^{v+2} - 1)(z^{v+3} - 1)(z^{v+4} - 1)(z^{2v+5} - 1)(z^{2I+1} - 1)}{z^{3v+I+2}(z^2 - 1)(z^3 - 1)(z^4 - 1)(z^5 - 1)}.$$

This expression has been derived by one of us (AAR) in [9]. Therein, results for several values of v and I ($0 \leq I \leq 11, 0 \leq v \leq 10$) have been given. The nice feature of this expression is that the function F has no pole in $z = 1$, this value of z being a zero for the numerator. Therefore, it is very easy to handle for any pair (v, I) , by applying the famous residue theorem of Cauchy. This expression is the starting point for our derivation of an analytical expression for $d_v(I)$.

We shall pursue this goal performing three steps: (a) express the fraction $[(1 - z^2)(1 - z^3)(1 - z^4)(1 - z^5)]^{-1}$ as a series in z of positive powers; (b) separate the singular part, denoted by G , from F , the holomorphic rest giving a vanishing contribution to $d_v^{(3)}(I)$; (c) calculate the residue for G .

To begin with, let us calculate the coefficients for the following expansion, considered for $|z| < 1$:

$$F_{k_1 k_2 k_3 k_4}(z) \equiv \frac{1}{(1 - z^{k_1})(1 - z^{k_2})(1 - z^{k_3})(1 - z^{k_4})} = \sum_{n=0}^{\infty} N_{k_1 k_2 k_3 k_4}(n) z^n. \tag{2.9}$$

Writing the above series as a product of four series associated with the simple fractions corresponding to the four factors appearing in the denominator in the above equation one easily obtains that $N_{k_1 k_2 k_3 k_4}(n)$ is nothing but the number of solutions (a, b, c, d) of the following equation:

$$k_1 a + k_2 b + k_3 c + k_4 d = n, \tag{2.10}$$

with a, b, c, d non-negative integer numbers. The number of solutions for this equation with four unknown positive integer numbers can be related to the number of solutions for an equation having only two non-negative integer unknowns

$$N_{k_1 k_2 k_3 k_4}(n) = \sum_{r=0}^n N_{k_1 k_2}(n - r) N_{k_3 k_4}(r). \tag{2.11}$$

Here $N_{k_1 k_2}(r)$ denotes the number of non-negative integer numbers solutions (a, b) for the equation

$$k_1 a + k_2 b = r. \tag{2.12}$$

For our purposes one needs to know only the functions with the particular indices $(k_1, k_2) = (1, k), (k, k + 1)$, i.e $N_{1,k}(r)$ and $N_{k,k+1}(r)$. For the first function $N_{1,k}(r)$, one obviously obtains

$$N_{1,k}(r) = \left[\frac{r}{k} \right] + 1. \tag{2.13}$$

For the other set of values $k_1 = k, k_2 = k + 1$, equation (2.12) becomes

$$ka + (k + 1)b = r, \tag{2.14}$$

which in its turn can be written as

$$ku + b = r, \tag{2.15}$$

where $u = a + b$. Taking into account the inequality $0 \leq b \leq u$ one obtains

$$\frac{r}{k + 1} \leq u \leq \frac{r}{k}, \tag{2.16}$$

and therefore

$$N_{k,k+1}(r) = \left[\frac{r}{k} \right] - \left[\frac{r}{k + 1} \right] + \chi \left(\frac{r}{k + 1} \right), \tag{2.17}$$

where $\chi(x) = 1$ if x is integer, and $\chi(x) = 0$ if x is noninteger. $\theta(x)$ denotes the step function defined as $\theta(x) = 1$ for $x \geq 0$ and $\theta(x) = 0$ for $x < 0$. Then, $\chi(x) = \theta([x] - x)$. In this way analytical expressions for the coefficients N_{k_1, k_2, k_3, k_4} of interest are obtained:

$$\begin{aligned} N_{2314}(n) &= \sum_{r=0}^n \left\{ \left[\frac{r}{3} \right] - \left[\frac{r}{4} \right] + \chi \left(\frac{r}{4} \right) \right\} \left\{ \left[\frac{n-r}{2} \right] + 1 \right\}, \\ N_{2315}(n) &= \sum_{r=0}^n \left\{ \left[\frac{r}{2} \right] - \left[\frac{r}{3} \right] + \chi \left(\frac{r}{3} \right) \right\} \left\{ \left[\frac{n-r}{5} \right] + 1 \right\}, \\ N_{2345}(n) &= \sum_{r=0}^n \left\{ \left[\frac{r}{2} \right] - \left[\frac{r}{3} \right] + \chi \left(\frac{r}{3} \right) \right\} \left\{ \left[\frac{n-r}{4} \right] - \left[\frac{n-r}{5} \right] + \chi \left(\frac{n-r}{5} \right) \right\}. \end{aligned} \tag{2.18}$$

In what follows we shall use the abbreviations

$$\begin{aligned} A(n) &= \frac{\theta(n)}{2} N_{2314}(n), \\ B(n) &= \frac{\theta(n)}{2} N_{2315}(n), \\ C(n) &= \frac{\theta(n)}{2} N_{2345}(n). \end{aligned} \tag{2.19}$$

Next we take account of expansion (2.9) for the particular indices $(k_1, k_2, k_3, k_4) = (2, 3, 4, 5)$ and by brute calculations we write the function F as a sum of a holomorphic function, which does not contribute to the integral (2.8), and a function G having poles in $z = 0$. The expression for G is

$$\begin{aligned} G(z) &= \frac{1}{z} \left(\frac{\theta(v - I - 3)}{z^{v-I-3}} - \frac{\theta(v + I - 2)}{z^{v+I-2}} + \frac{\theta(I - v - 7)}{z^{I-v-7}} \right) F_{2314}(z) \\ &+ \frac{1}{z} \left(\frac{\theta(2v + I)}{z^{2v+I}} - \frac{\theta(2v - I - 1)}{z^{2v-I-1}} - \frac{\theta(I - 2v - 10)}{z^{I-2v-10}} \right) F_{2315}(z) \\ &+ \frac{1}{z} \left(\frac{\theta(3v - I)}{z^{3v-I}} - \frac{\theta(3v + I + 1)}{z^{3v+I+1}} + \frac{\theta(I - 3v - 14)}{z^{I-3v-14}} \right) F_{2345}(z). \end{aligned} \tag{2.20}$$

Using this expression, the residue for F is readily obtained and the final result for the degeneracy $d_v^{(3)}(I)$, characterizing the R_7 irreducible representation, is

$$\begin{aligned} d_v^{(3)}(I) &= A(v - I - 3) - A(v + I - 2) + A(I - v - 7) + B(2v + I) - B(2v - I - 1) \\ &- B(I - 2v - 10) + C(3v - I) - C(3v + I + 1) + C(I - 3v - 14), \end{aligned} \tag{2.21}$$

where the functions A , B and C were defined in equation (2.19).

3. Degeneracy of the quadrupole boson states

The case of quadrupole degeneracy may be treated in a similar way to that of octupole degeneracy. Indeed, the character of an irreducible representation is defined by an equation which, formally, is identical to equation (2.1), with the difference that now all vectors involved have two components. Indeed, for $R_5, (L_1, L_2) = (3/2, 1/2)$, and the highest weight vector is $(S_1, S_2) = (v, 0)$, where v denotes the seniority quantum number for the quadrupole boson system. The reduction from R_5 to R_3 is achieved by setting

$$\frac{\varphi_1}{2} = \varphi_2 \equiv \varphi. \tag{3.1}$$

The final expression for $\chi_v^{(2)}$ can be written as a ratio of two determinants:

$$\chi_v^{(2)}(\varphi) = \frac{\Delta(v)}{\Delta(0)},$$

where

$$\Delta(v) = \det \begin{pmatrix} e^{i\varphi(2v+3)} - e^{-i\varphi(2v+3)} & e^{i\varphi} - e^{-i\varphi} \\ e^{i\varphi(v+\frac{3}{2})} - e^{-i\varphi(v+\frac{3}{2})} & e^{i\frac{1}{2}\varphi} - e^{-i\frac{1}{2}\varphi} \end{pmatrix}. \tag{3.2}$$

The R_5 degeneracy caused by the reduction $R_5 \supset R_3$ is further expressed as

$$d_v^{(2)}(I) = \int_0^{2\pi} \chi_I^*(\varphi) \chi_v^{(2)}(\varphi) \rho(\varphi) d\varphi, \tag{3.3}$$

where χ_I and ρ are the functions defined by equations (2.3) and (2.5), respectively. Changing the variable $z = e^{i\varphi}$, $d_v^{(2)}(I)$ is expressed as a contour integral

$$d_v^{(2)}(I) = \frac{i}{4\pi} \int_{|z|=1} \frac{1}{z^{2v+I+2}} \frac{(z^{v+1}-1)(z^{v+2}-1)(z^{2v+3}-1)(z^{2I+1}-1)}{(z^2-1)(z^3-1)} dz. \quad (3.4)$$

Following the same procedure as in the previous section, we perform the expansion

$$F_{k_1 k_2}(z) \equiv \frac{1}{(1-z^{k_1})(1-z^{k_2})} = \sum_{n=0}^{\infty} N_{k_1 k_2}(n) z^n. \quad (3.5)$$

The needed expansion coefficients have already been calculated (see equations (2.13), (2.17)):

$$N_{23}(n) = \left[\frac{n}{2} \right] - \left[\frac{n}{3} \right] + \chi\left(\frac{n}{3}\right), \quad N_{13}(n) = \left[\frac{n}{3} \right] + 1. \quad (3.6)$$

The singular part of the integrand of equation (3.4) is

$$G(z) = \frac{1}{z} \left(\frac{\theta(I-2v-5)}{z^{I-2v-5}} + \frac{\theta(2v-I)}{z^{2v-I}} - \frac{\theta(2v+I+1)}{z^{2v+I+1}} \right) F_{13}(z) \\ + \frac{1}{z} \left(\frac{\theta(v+I)}{z^{v+I}} - \frac{\theta(v-I-1)}{z^{v-I-1}} - \frac{\theta(I-v-3)}{z^{v-I-3}} \right) F_{23}(z). \quad (3.7)$$

With these details the residue for the function G is readily calculated and the final result for multiplicity is

$$d_v^{(2)}(I) = P(I-2v-5) + P(2v-I) - P(2v+I+1) \\ + Q(v+I) - Q(v-I-1) - Q(I-v-3), \quad (3.8)$$

where Q and P denote

$$Q(n) = \frac{1}{2}\theta(n)N_{13}(n), \quad P(n) = \frac{1}{2}\theta(n)N_{23}(n). \quad (3.9)$$

Before closing this section we remark that the integral representation for the R_7 and R_5 symmetry groups can be written in a unified manner:

$$d_v^{(l)}(I) = \frac{i}{4\pi} \int_{|z|=1} \frac{(z^{2I+1}-1)(z^{2v+2l-1}-1) \prod_{k=1}^{2l-2} (z^{v+k}-1)}{z^{lv+I+2} \prod_{k=1}^{2l-2} (z^{k+1}-1)} dz, \quad (3.10)$$

with $l = 2$ for R_5 and $l = 3$ for R_7 .

4. The general case of R_{2l+1} degeneracy

Note that we started by treating first the octupole state degeneracy. The reason is that the procedure uses the contour integral expression for the states, degeneracy derived for the first time [9, 10] in connection with the octupole degrees of freedom. Moreover, since the quadrupole shape coordinates are widely used for describing the collective properties in nuclear systems, we have applied the method, formulated in section 2, also to this particular type of state. Noteworthy is the fact that the degeneracies for the quadrupole and octupole states could be written in a unified fashion (3.10). This result poses a challenge to us to prove that equation (3.10) gives, in fact, the multiplicity for the reduction $R_{2l+1} \supset R_3$. As a matter of fact this is the objective of this section.

For the group R_{2l+1} the character function is

$$\chi_{lv}(\varphi_1, \varphi_2, \dots, \varphi_l) = \frac{\det(e^{i\varphi_m K_n} - e^{-i\varphi_m K_n})_{1 \leq m, n \leq l}}{\det(e^{i\varphi_m S_n} - e^{-i\varphi_m S_n})_{1 \leq m, n \leq l}}, \quad (4.1)$$

where the vector $K = (K_1, \dots, K_l)$ is the sum of the root vector $L = (L_1, \dots, L_l)$ and the highest weight vector $S = (S_1, \dots, S_l)$ defined by

$$L_k = l - k + \frac{1}{2}, \quad S_k = v\delta_{k1}. \tag{4.2}$$

The reduction $R_{2l+1} \supset R_3$ is achieved by the restrictions

$$\varphi_k = k\varphi, \quad k = 1, 2, \dots, l. \tag{4.3}$$

Following the procedure of the previous sections, the degeneracy is defined as the coefficients of the expansion of $\chi_I(\varphi)$ in terms of $\chi_{lv}(\varphi)$:

$$d_v^{(l)} = \int_0^{2\pi} \chi_I^*(\varphi)\chi_{lv}(\varphi)\rho(\varphi) d\varphi. \tag{4.4}$$

Changing the variable φ to $z = e^{i\varphi}$ one obtains

$$d_v^{(l)}(I) = \frac{i}{4\pi} \int_{|z|=1} \frac{(z^{2l+1} - 1)U_{lv}(z)}{z^{lv+l+2}V_{lv}(z)} dz, \tag{4.5}$$

where U_{lv} and V_{lv} are the following polynomials in z :

$$U_{lv}(z) = (z^{2v+2l-1} - 1) \prod_{k=1}^{2l-2} (z^{v+k} - 1), \quad V_{lv}(z) = \prod_{k=1}^{2l-2} (z^{k+1} - 1). \tag{4.6}$$

Let us denote by $D_{vl}(m)$ and $N_l(n)$ the coefficients for the U polynomial and the Taylor expansion associated with $1/V_{lv}$

$$U_{lv}(z) = \sum_{m \geq 0} D_{lv}(m)z^m, \quad \frac{1}{V_{lv}(z)} = \sum_{n=0}^{\infty} N_{(l)}(n)z^n. \tag{4.7}$$

Here $N_l(n)$ denotes the number of solutions (n_1, \dots, n_{2l-2}) of the following equation:

$$\sum_{k=1}^{2l-2} (k+1)n_k = n, \tag{4.8}$$

with n_1, \dots, n_{2l-2} non-negative integer numbers. For the particular cases of $l = 2, 3$, we have been able to provide analytical solutions for $N_l(n)$. It is an open question whether this is possible for the general case. However, for an arbitrary l , recursive equations for $N_l(n)$ are obtainable. Note that inserting the expressions for the U and V polynomials in equation (4.5), one obtains the unifying expression (3.10) for the quadrupole and octupole degeneracies.

Inserting expansion (4.7) in equation (4.5), the residue can be easily calculated. The final expression for $d_v^{(l)}$ is

$$d_v^{(l)}(I) = \sum_{n \geq 0} N_l(n)[\theta(lv - I - n)D_{lv}(lv - I - n) - \theta(lv + I - n + 1)D_{lv}(lv + I - n + 1)]. \tag{4.9}$$

Summarizing the results obtained in this paper we can assert that the multiplicity of a irreducible representation (I) of the group R_3 in a given irreducible representation (v) of the group R_{2l+1} is obtained by performing the following steps:

- (a) the character of (v) representation is expanded in terms of the characters of IR representations (I) .
- (b) The expansion coefficients are written as a contour integral.
- (c) The contour integral is performed by making use of the Cauchy theorem.

The group reduction $R_{2l+1} \subset R_3$ reflects itself into the restriction of the character support given by equation (4.3). Such a reduction is described in detail in [16] for the general case and in [6] and [9] for the quadrupole and octupole cases, respectively. A different embedding of R_3 into R_7 was proposed in [11] where the components of the angular momentum operator, acting on the space of octupole shape coordinates and the corresponding conjugate momenta, are expressed as a linear combination of the R_7 generators. Since the dimensions of the irreducible representations do not depend on the specific realization of the space on which group elements are to act, the degeneracies obtained by the two embeddings are identical. Actually this can easily be checked by comparing the results tabulated in [9, 11], obtained by using different embeddings.

5. Summary

The main results of the present paper can be summarized as follows: based on their integral representations (see equations (2.8) and (3.4)), the R_7 and R_5 irreducible representation degeneracies are analytically derived and given by equations (2.21) and (3.8), respectively. The generalization to the group R_{2l+1} has been presented in section 4 and a compact formula for the corresponding degeneracy $d_v^{(l)}(I)$ was derived (see equation (4.9)).

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